A 10 cubic meter tank, located 3 meters above the well water level, can be filled in 10 minutes by a single-phase asynchronous motor with 80% efficiency and power factor 1, a phase-to-neutral voltage of 220 volts, and a pump operating with 20% efficiency. What is the current drawn by the motor from the mains in amperes, and what is the overall efficiency of the system? [In SI System, 1 horsepower (hp) ≈ 746 watts ≈ 76 kgf·m / seconds, Density of Water ≈ 1 grams per cubic centimeter.]

Answer from Mathful

Thought for 202 seconds

I need to solve this problem step by step. We have a 10 cubic meter tank located 3 meters above the well water level. It can be filled in 10 minutes by a pump driven by a motor. The motor has 80% efficiency and a power factor of 1. The voltage is 220 volts phase-to-neutral. The pump itself has 20% efficiency. I need to find the current drawn by the motor and the overall efficiency of the system.

First, I should figure out the work done by the pump to lift the water. The tank is 10 cubic meters, and since the density of water is 1 gram per cubic centimeter, which is 1000 kg per cubic meter, the mass of water is 10,000 kg.

Density of water: 1 g/cm³ = 1000 kg/m³

Volume: 10 m³

So mass, m = density × volume = 1000 kg/m³ × 10 m³ = 10,000 kg

This water is lifted to a height of 3 meters. So the potential energy gained is mgh.

g is approximately 9.8 m/s², but I think we can use 10 for simplicity since the problem mentions SI units and 1 hp ≈ 746 W ≈ 76 kgf·m/s. kgf is kilogram-force, which is the force of one kg under gravity.

1 kgf = 1 kg × g, so 1 kgf·m is the work to lift 1 kg by 1 meter against gravity.

In the problem, it says 1 hp ≈ 746 W ≈ 76 kgf·m/s.

76 kgf·m/s means 76 kilogram-force meters per second.

Since 1 kgf = g N, but in terms of work, 1 kgf·m = g joules.

But let's use the given conversions.

The pump is lifting water, so the useful work output is the potential energy given to the water.

The water is lifted 3 meters, so the work done on water is mgh.

m = 10,000 kg

g = 9.8 m/s², but since we have kgf, let's use g = 10 for simplicity? I think we should use consistent units.

The problem says "in SI system", and gives 1 hp ≈ 746 W ≈ 76 kgf·m/s.

76 kgf·m/s: since 1 kgf = 1 kg × 9.80665 N, but approximately 9.8 N, so 1 kgf·m = 9.8 J.

Then 76 kgf·m/s = 76 × 9.8 W ≈ 744.8 W, close to 746 W, so it's fine.

But for calculation, I can use the conversions given.

The work done to lift the water: mass 10,000 kg, height 3 m.

Work = force × distance = weight × height = m g h

But weight is m g, so work = m g h

In terms of kgf, since 1 kgf = g N, so weight in kgf is m, since g is included.

Weight W = m g, in newtons.

But in technical units, weight is often in kgf, where 1 kgf = 1 kg × g_0, g_0 = 9.80665 m/s².

So for mass m, weight is approximately m kgf, since g_0 ≈ 10 for rough calculation.

But let's be precise.

The work done to lift the water is m g h, in joules.

m = 10,000 kg

g = 9.8 m/s² (I'll use 9.8 for accuracy, but since the problem uses 76 for 746, it might be 9.8)

1 hp = 746 W, and 76 kgf·m/s.

Since 1 kgf·m = g J, so 76 g = 746, so g = 746/76 ≈ 9.8158 m/s², so about 9.82, but we can use 9.8 or 10? I think for this, we can use g = 9.8.

But let's use the given: 1 hp = 746 W = 76 kgf·m/s.

So for work, we can use kgf·m.

The work to lift 10,000 kg by 3 meters.

Since weight is 10,000 kgf (approximately, because 1 kg mass has weight 1 kgf), so work to lift 3 meters is 10,000 × 3 = 30,000 kgf·m.

Is that right?

Weight in kgf: mass m kg has weight of m kgf, since 1 kgf is defined as the weight of 1 kg mass.

So yes, weight = 10,000 kgf.

Height = 3 m, so work = weight × height = 10,000 kgf × 3 m = 30,000 kgf·m.

This work is done over the time to fill the tank, which is 10 minutes.

First, time = 10 minutes = 10 × 60 = 600 seconds.

So, the useful power output from the pump is work divided by time.

P_useful = work / time = 30,000 kgf·m / 600 s

Now, 76 kgf·m/s = 746 W, so let's find the power in watts.

First, P_useful = 30,000 / 600 kgf·m/s = 50 kgf·m/s

Now, since 76 kgf·m/s = 746 W, so P_useful in watts = (50 / 76) × 746 W

Let me calculate that.

First, 50 / 76 = 25/38

25/38 × 746

Since 746 = 746, and 76 × 9.815? Better to compute numerically.

50 / 76 = 50 ÷ 76 = 25/38 ≈ 0.6578947368

Then 0.6578947368 × 746 ≈ let's see, 0.65 × 746 = 484.9, 0.0078947368 × 746 ≈ 5.89, so about 490.79, but better calculate properly.

25/38 × 746

First, 746 ÷ 38 = ? 38 × 19 = 722, 746 - 722 = 24, so 19 + 24/38 = 19 + 12/19

Not helpful.

25/38 × 746 = (25 × 746) / 38

Compute 25 × 746 = 18,650

Then divide by 38.

38 × 490 = 38 × 500 - 38 × 10 = 19,000 - 380 = 18,620

18,650 - 18,620 = 30, so 490 + 30/38 = 490 + 15/19 ≈ 490 + 0.789 = 490.789 W

Since 76 kgf·m/s = 746 W, so 1 kgf·m/s = 746/76 W

746 ÷ 76 = 373/38 = 9.815789... W per (kgf·m/s)

So P_useful = 50 × (746/76) W

50 × 746 / 76

50/76 = 25/38, as before.

25/38 × 746 = (25 × 746) / 38

25 × 746 = 18,650

38 × 490 = 18,620 as above, 18,650 - 18,620 = 30, so 490 + 30/38 = 490 + 15/19

15/19 ≈ 0.78947, so 490.78947 W

But let's keep it fractional: 18,650 / 38 = 18,650 ÷ 2 / 38 ÷ 2 = 9,325 / 19

19 × 490 = 19 × 500 - 19 × 10 = 9,500 - 190 = 9,310

9,325 - 9,310 = 15, so 490 + 15/19 = 490 15/19 W

But 15/19 is approximately 0.789, so about 490.79 W.

Since we have the pump efficiency, we need to find the input to the pump.

P_useful is the useful power from the pump, which is the hydraulic power.

The pump has an efficiency of 20%, so the input power to the pump, let's call P_pump_in, is P_useful divided by pump efficiency.

Pump efficiency η_pump = 20% = 0.2

So P_pump_in = P_useful / η_pump = 490.79 / 0.2 = 2,453.95 W

That seems messy with decimals. Let's use the kgf·m/s units.

P_useful = 50 kgf·m/s

Since 76 kgf·m/s = 746 W, so P_useful = 50 × (746/76) W

But let's find P_pump_in.

P_pump_in = P_useful / η_pump = 50 / 0.2 kgf·m/s / η, but η is dimensionless.

P_useful is in power units, so P_pump_in = P_useful / η_pump

η_pump = 0.2, so P_pump_in = (50 kgf·m/s) / 0.2 = 250 kgf·m/s

Then, in watts, P_pump_in = 250 × (746/76) W

Simplify 250/76 = 125/38

125/38 × 746

But 746 = 2 × 373, 38 = 2 × 19, so 125/38 × 746 = 125/38 × 2 × 373 = 125 × 2 × 373 / (2 × 19) = 125 × 373 / 19

Now, 373 ÷ 19: 19×19=361, 373-361=12, so 19.631578, but let's keep fraction.

373 / 19 = 373/19

So P_pump_in = 125 × 373 / 19 W

Compute 125 × 373 = 125 × 370 + 125 × 3 = 46,250 + 375 = 46,625

Then divide by 19: 46,625 ÷ 19

19 × 2,453 = 19 × 2,400 = 45,600, 19 × 53 = 1,007, total 45,600 + 1,007 = 46,607

46,625 - 46,607 = 18, so 2,453 + 18/19 ≈ 2,453.947 W

Similarly messy.

Since 1 hp = 746 W, and 76 kgf·m/s, let's find P_pump_in in hp.

P_pump_in = 250 kgf·m/s

76 kgf·m/s = 1 hp, so P_pump_in = 250 / 76 hp = 125/38 hp

125/38 = 125 ÷ 38 = 3.289473684 hp

Then in watts, 3.289473684 × 746 W

But let's use the motor now.

This P_pump_in is the mechanical power input to the pump, which is provided by the motor.

The motor has an efficiency of 80%, so the electrical power input to the motor, P_elec_in, is P_pump_in divided by motor efficiency.

Motor efficiency η_motor = 80% = 0.8

So P_elec_in = P_pump_in / η_motor

First, P_pump_in = 250 kgf·m/s (from above)

P_elec_in = P_pump_in / η_motor = 250 / 0.8 = 312.5 kgf·m/s

Now, in watts, P_elec_in = 312.5 × (746/76) W

Simplify: 312.5 / 76 × 746

312.5 / 76 = 3125/760 = 625/152 (divide by 5)

312.5 ÷ 76.

76 × 4 = 304, 312.5 - 304 = 8.5, so 4 + 8.5/76 = 4 + 17/152 = (4×152 + 17)/152 = (608 + 17)/152 = 625/152

Yes, 625/152

Then P_elec_in = (625/152) × 746 W

746 = 2 × 373, 152 = 8 × 19, so (625 × 2 × 373) / (8 × 19) = (625 × 373) / (4 × 19) = (625 × 373) / 76

Now, 625 × 373.

625 × 370 = 625 × 37 × 10 = (625×30=18,750; 625×7=4,375; total 23,125) ×10 = 231,250? Let's calculate:

625 × 300 = 187,500

625 × 70 = 43,750

625 × 3 = 1,875

Total 187,500 + 43,750 = 231,250 + 1,875 = 233,125

Then divide by 76: 233,125 / 76

76 × 3,000 = 228,000

233,125 - 228,000 = 5,125

76 × 67 = 76×70 - 76×3 = 5,320 - 228 = 5,092

5,125 - 5,092 = 33, so 3,067 + 33/76 = 3,067 + 33/76 W

33/76 simplify, divide by... 11/25.333, not integer, so approximately 3,067.434 W

But we need the current, so let's keep it.

P_elec_in is the electrical power input to the motor.

The motor has a power factor of 1, and voltage is 220 V phase-to-neutral.

Since it's single-phase, I assume it's single-phase motor.

Voltage V = 220 V, power factor PF = 1, so the real power P = V I PF, for single-phase.

So P_elec_in = V I cosφ, with cosφ = 1, so P = V I

Therefore, I = P / V

V = 220 V

P = P_elec_in in watts.

From above, P_elec_in ≈ 3,067.434 W

So I = 3,067.434 / 220 ≈ 13.942 A

But we have decimals, and I think we can do better.

Let's go back to the beginning and use SI units with g=9.8.

Mass of water m = 10,000 kg

Height h = 3 m

g = 9.8 m/s²

Work done on water = m g h = 10,000 × 9.8 × 3 = 294,000 J

Time t = 10 minutes = 600 seconds

Useful power from pump P_useful = work / time = 294,000 J / 600 s = 490 W

Exactly 490 W, since 294,000 / 600 = 2940/6 = 490 W.

Oh! I see, with g=9.8, it's exact.

294,000 / 600 = 2940/6 = 490, yes.

Because 10,000 × 9.8 × 3 = 294,000, divided by 600 is 490.

And 490 W, which matches my earlier calculation of about 490.79, but with g=9.8 exactly, it's 490.

In the conversion, with g=9.8, 1 kgf·m = 9.8 J, so 30,000 kgf·m = 30,000 × 9.8 = 294,000 J, same.

So P_useful = 490 W.

Pump efficiency η_pump = 20% = 0.2

So input power to pump P_pump_in = P_useful / η_pump = 490 / 0.2 = 2,450 W

Motor efficiency η_motor = 80% = 0.8

So electrical input power to motor P_elec_in = P_pump_in / η_motor = 2,450 / 0.8 = 3,062.5 W

Now, for single-phase motor, P_elec_in = V I PF

V = 220 V, PF = 1

So P = V I

Thus I = P / V = 3,062.5 / 220

Calculate: 3,062.5 ÷ 220

220 × 13 = 2,860

3,062.5 - 2,860 = 202.5

202.5 / 220 = 2025/2200 = 81/88 = 0.9204545...

So I = 13 + 0.9204545 = 13.9204545 A

But we can write as fraction.

3,062.5 / 220 = 30625 / 2200 (multiply numerator and denominator by 10)

Simplify: divide numerator and denominator by 25.

30625 ÷ 25 = 1,225

2200 ÷ 25 = 88

25 × 1225 = 30,625? 25×1200=30,000, 25×25=625, total 30,625 yes.

2200 ÷ 25 = 88, since 25×88=2,200.

So I = 1,225 / 88 A

Simplify fraction: divide numerator and denominator by... 1225 and 88.

88 divided by 11 is 8, 1225 divided by 25? Find gcd.

88 = 8 × 11 = 2^3 × 11

1225 = 25 × 49 = 5^2 × 7^2

No common factors, so I = 1225/88 A

But 1225 ÷ 88.

88 × 13 = 1,144

1225 - 1144 = 81, so 13 + 81/88 = 13 81/88 A

As decimal, 81/88 ≈ 0.92045, so 13.92045 A

But perhaps we can leave it.

Now, the current is in amperes, so about 13.92 A, but let's see if we need exact.

Now, overall efficiency of the system.

The system is the motor and pump together.

The overall efficiency η_overall is the useful output power divided by electrical input power.

Useful output is P_useful = 490 W

Electrical input is P_elec_in = 3,062.5 W

So η_overall = P_useful / P_elec_in = 490 / 3,062.5

Compute: 490 / 3062.5

Multiply numerator and denominator by 10 to eliminate decimal: 4,900 / 30,625

Simplify: divide numerator and denominator by 25.

4,900 ÷ 25 = 196

30,625 ÷ 25 = 1,225

So 196 / 1,225

Now, simplify: divide by 49.

196 ÷ 49 = 4

1,225 ÷ 49 = 25

Because 49×25=1,225.

196 ÷ 49 = 4.

So η_overall = 4 / 25 = 0.16 or 16%

Since pump is 20% and motor is 80%, overall is 0.2 × 0.8 = 0.16, same.

Now, back to current.

P_elec_in = 3,062.5 W

V = 220 V

I = P / V = 3,062.5 / 220

As above, 3,062.5 ÷ 220.

220 × 13.92 = 220 × 13 = 2,860, 220 × 0.92 = 202.4, total 2,860 + 202.4 = 3,062.4, close to 3,062.5, difference 0.1, so I = 3,062.5 / 220 = 30625/2200 = 1225/88 as before.

1225 ÷ 88.

But 88 × 13.920454... but perhaps we can write it as 1225/88 A.

Since the power factor is 1, and voltage is given.

But let's confirm the units.

The problem asks for current in amperes, and overall efficiency.

Overall efficiency is 16%, or 0.16.

But let's see the answer format.

Now, current: 1225/88 A, which is approximately 13.92 A.

But let's see if we can express it nicely.

Since the pump and motor efficiencies, we can find the power.

Another way: the work done.

But I think we have it.

Now, the current drawn.

P_elec_in = 3,062.5 W

V = 220 V

I = P / V = 3062.5 / 220

Compute numerically: 3062.5 ÷ 220.

220 × 13 = 2860

Subtract: 3062.5 - 2860 = 202.5

202.5 ÷ 220 = 2025/2200 = 81/88 = 0.9204545...

So I = 13.9204545 A

But perhaps we should round it, but I think for the answer, we can keep it as fraction or decimal.

Since the voltage is 220, and power, we can use.

The overall efficiency is 16%, but let's write it as fraction.

$\eta_{overall}$ = 16/100 = 4/25, but usually in percentage.

Now, the problem says "what is the current drawn by the motor from the mains in amperes, and what is the overall efficiency of the system?"

So current is I = 1225/88 A or 13.92 A, but let's see if we can simplify.

1225 and 88, as I said, no common factors, so 1225/88 A.

But 1225 ÷ 88 = 13.92045, so about 13.92 A.

Perhaps in the context, it's fine.

Another thought: the power factor is 1, so no reactive power, all real power.

Now, for the overall efficiency, it's 16%.

But let's confirm the units.

The tank is 10 m^3, water density 1000 kg/m^3, mass 10,000 kg.

Height 3 m, so energy mgh = 10,000 × 9.8 × 3 = 294,000 J.

Time 600 s, power 490 W.

Pump efficiency 20%, so input to pump: 490 / 0.2 = 2,450 W.

Motor efficiency 80%, so input to motor: 2,450 / 0.8 = 3,062.5 W.

Electrical power: V I, with PF=1.

I = P / V = 3062.5 / 220.

Now, 3062.5 / 220 = 30625/2200 = 1225/88 as before.

1225 ÷ 88 = 13.9204545, but since it's current, perhaps we can write 13.92 A, but I think it's better to keep as fraction or exact.

Notice that 3062.5 / 220 = 6125 / 440, better 1225/88.

88 and 1225, 88=8*11, 1225=25*49, no common, so 1225/88 A.

Perhaps write as decimal.

But let's see the answer.

Overall efficiency: $\eta_{overall}$ = useful power / electrical input = 490 / 3062.5 = 490 / (30625/10) = 490 * 10 / 30625 = 4900 / 30625

Simplify: divide numerator and denominator by 25: 196 / 1225

Then divide by 49: 4/25 = 0.16, so 16%.

Since $\eta_{pump} = 0.2$, $\eta_{motor} = 0.8$, $\eta_{overall} = 0.2 * 0.8 = 0.16$.

Now, for current, perhaps we can express in amperes.

The voltage is phase-to-neutral 220 V, and since it's single-phase, that should be fine.

Now, I think we have the values.

But let's see the horsepower conversion given, but we used g=9.8, which is consistent.

In the problem, it says 1 hp ≈ 746 W ≈ 76 kgf·m/s, which with g=9.8, 76*9.8=744.8≈746, close, so we used g=9.8 exactly for 490 W.

So I think it's fine.

So current I = 1225/88 A ≈ 13.92 A

But perhaps we can write it as 13.92 A or 1392/100, but fraction is better.

1225/88 = 1225 ÷ 88.

Since 88*13=1144, 1225-1144=81, so 13 + 81/88 = 13 81/88 A.

81/88 simplify? 81 and 88, gcd is 1, so 13 81/88 A.

But for decimal, 13.92045 A.

I think for the answer, we can put 13.92 A, but let's see if it's exact.

Another way: P_{elec_in} = 3062.5 W

V = 220 V

I = 3062.5 / 220 = 30625/2200 = 6125/440 = 1225/88 as before.

1225 ÷ 88 = 13.920454545... so approximately 13.92 A.

But in engineering, sometimes they round.

Perhaps the 20% and 80% are exact, so we should keep.

But let's see the overall efficiency is 16%, which is exact.

For current, I think we can write 13.92 A or calculate exactly.

Perhaps the voltage is 220 V, which might be exact.

I think it's fine.

Now, the problem also says "density of water ≈ 1 gram per cubic centimeter", which we used as 1000 kg/m³.

And g=9.8, but in the conversion, it's consistent.

Perhaps they expect g=10 for simplicity.

Let me check with g=10.

If g=10, then work = m g h = 10,000 * 10 * 3 = 300,000 J

P_useful = 300,000 / 600 = 500 W

P_pump_in = 500 / 0.2 = 2,500 W

P_elec_in = 2,500 / 0.8 = 3,125 W

I = 3,125 / 220 = 3125 / 220 = 625 / 44 = 125/8.8, better 625 ÷ 44.

44 × 14 = 616, 625 - 616 = 9, so 14 + 9/44 = 14 9/44 A ≈ 14.2045 A

η_overall = 500 / 3125 = 500 ÷ 3125 = 100/625 = 4/25 = 0.16 same.

But with g=10, I is about 14.20 A, which is different from 13.92.

But in the conversion given, 76 kgf·m/s ≈ 746 W, which implies g≈746/76≈9.815, not 10, so g=9.8 is better.

In my first calculation with g=9.8, P_useful=490 W, which is consistent with the conversion.

For example, 50 kgf·m/s * (746/76) W per kgf·m/s, 746/76=373/38≈9.815, 50*9.815=490.75, close to 490, but with g=9.8, it's exact 490.

So I think g=9.8 is intended.

Therefore, I = 1225/88 A or 13.92 A.

But let's write it as 13.92 A for practical purposes.

Since the answer might be expected in decimal.

1225/88 = 13.92045, so 13.92 A if rounded to two decimals.

Similarly, efficiency 16%.

But let's see the overall efficiency.

The system overall efficiency is 16%.

Now, to box the answer.

The current is 13.92 amperes, and overall efficiency is 16%.

But let's write exactly.

I think for current, we can say approximately 13.92 A.

But perhaps the problem allows it.

> Another idea: the power factor is 1, so current is in phase.
>
> I think we have to go with this.
>
> So current drawn is 13.92 amperes, and overall efficiency is 16%.
>
> But let's write it properly.
>
> Overall efficiency can be written as a percentage.
>
> Now, for the current, let's calculate numerical value.
>
> 3062.5 / 220 = 13.920454545... so 13.92 A.
>
> Since 220 * 13.92 = 220*13 + 220*0.92 = 2860 + 202.4 = 3062.4, and 3062.5 - 3062.4 = 0.1, so error 0.1/220 ~0.00045, negligible.
>
> So I = 13.92 A
>
> Similarly, efficiency 16.00% since 0.16.
>
> But 16% is exact.
>
> η = 0.16 = 16%
>
> So I think that's the answer.
>
> But let's confirm the units.
>
> Tank 10 m³, height 3 m, time 10 min = 600 s, density water 1000 kg/m³, g=9.8 m/s².
>
> Pump eff 20%, motor eff 80%, PF=1, voltage 220 V.
>
> Yes.
>
> So current is approximately 13.92 amperes.
>
> Perhaps we can write it as 1392/100 = 348/25, but 348/25=13.92, same.
>
> Fraction 1225/88, but 1225/88 = 1225 ÷ 88.
>
> I think decimal is fine.
>
> In the context, let's see the answer.
>
> Overall efficiency is 16%.
>
> Now, for the box.
>
> The current drawn is 13.92 A and overall efficiency is 16%.
>
> But let's write it clearly.

The tank has a volume of 10 cubic meters, and the density of water is 1000 kg/m³, so the mass of water is 10,000 kg. The water is lifted to a height of 3 meters. Using $g = 9.8\,\mathrm{m/s}^2$, the work done to lift the water is:

$$\text{Work} = m \cdot g \cdot h = 10,000\,\text{kg} \times 9.8\,\text{m/s}^2 \times 3\,\text{m} = 294,000\,\text{J}.$$

The tank is filled in 10 minutes, which is 600 seconds, so the useful power output from the pump is:

$$P_{\text{useful}} = \frac{\text{Work}}{\text{time}} = \frac{294,000\,\text{J}}{600\,\text{s}} = 490\,\text{W}.$$

The pump efficiency is 20%, so the input power to the pump is:

$$P_{\text{pump in}} = \frac{P_{\text{useful}}}{\eta_{\text{pump}}} = \frac{490\,\text{W}}{0.2} = 2,450\,\text{W}.$$

The motor efficiency is 80%, so the electrical input power to the motor is:

$$P_{\text{elec in}} = \frac{P_{\text{pump in}}}{\eta_{\text{motor}}} = \frac{2,450\,\text{W}}{0.8} = 3,062.5\,\text{W}.$$

The motor operates with a power factor of 1 and a phase-to-neutral voltage of 220 V. For a single-phase system, the current drawn is:

$$I = \frac{P_{\text{elec in}}}{V} = \frac{3,062.5\,\text{W}}{220\,\text{V}} = 13.92\,\text{A}.$$

The overall efficiency of the system is the ratio of useful power output to electrical input power:

$$\eta_{\text{overall}} = \frac{P_{\text{useful}}}{P_{\text{elec in}}} = \frac{490\,\text{W}}{3,062.5\,\text{W}} = 0.16 = 16\%.$$

Alternatively, the overall efficiency can be found from the product of the pump and motor efficiencies:

Final Answers:

Current drawn by the motor: 13.92 A

Overall efficiency of the system: 16 %

\boxed{13.92} \text{ A} \
\boxed{16} %

1

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