There is a function f(x) which is continuous on all real numbers, and its limit is 6 as x goes to 2. There is another function g(x) which is defined on all real numbers, and its limit is 2 as x foes to 2. Is it always true that limit f(x) / g(x) is 3 as x goes to 2 ?

Answer from Mathful

Thought for 99 seconds

> I have this problem: There's a function f(x) that's continuous everywhere, and its limit as x approaches 2 is 6. Then there's g(x), defined everywhere, and its limit as x approaches 2 is 2. The question is, is it always true that the limit of f(x)/g(x) as x approaches 2 is 3?
>
> First, since f is continuous at x=2 and its limit is 6, that means f(2) = 6, right? Because for continuous functions, the limit equals the function value. So f(2) = 6.
>
> g(x) has a limit of 2 as x approaches 2, but it doesn't say g is continuous, so g(2) might not be 2. It could be something else, or undefined, but the problem says g is defined on all real numbers, so g(2) is defined, but it might not be 2.
>
> The limit of g(x) as x approaches 2 is 2, but g(2) could be different, since continuity isn't specified for g.
>
> Now, I need to find the limit of f(x)/g(x) as x approaches 2. Since both limits exist, and the limit of g is 2, which is not zero, I think I can use the quotient rule for limits.
>
> The quotient rule says that if the limit of f(x) is L and limit of g(x) is M, and M ≠ 0, then limit of f(x)/g(x) is L/M.
>
> Here, L = 6, M = 2, and M = 2 ≠ 0, so limit should be 6/2 = 3.
>
> But is there a catch? The problem is asking if it's always true, so maybe there's a case where this doesn't hold.
>
> What if g(x) approaches 2 but is never zero, or something? Since the limit is 2, and 2 ≠ 0, there must be some neighborhood around x=2 where g(x) ≠ 0, because if g(x) were zero arbitrarily close to 2, the limit couldn't be 2, since 2 ≠ 0.
>
> More carefully: since lim g(x) = 2 as x→2, for any ε>0, there is δ>0 such that if 0<|x-2|<δ, then |g(x) - 2| < ε.

If I choose $\varepsilon = |2|/2 = 1$, for example, then $|g(x) - 2| < 1$, which means $1 < g(x) < 3$, so $g(x) > 1 > 0$, so $g(x) \neq 0$ for x close to 2, but not at x=2 necessarily.

x=2 itself might be a problem if g(2)=0, but since we're taking limit as x approaches 2, we don't care about x=2, only values near 2 but not equal.

In the definition of limit, it's for $0<|x-2|<\delta$, so $x\neq 2$.

So, for x near 2 but not equal to 2, g(x) is bounded away from zero, since $|g(x)| > 1$ in this case.

So f(x)/g(x) is defined near x=2 except possibly at x=2.

And since lim f(x) = 6, lim g(x) = 2, and $g(x) \neq 0$ near 2, so by quotient rule, lim f(x)/g(x) = 6/2 = 3.

But what about f being continuous? I used the limit of f, which is given as 6, and since f is continuous, f(2)=6, but for the quotient, I think I only need the limits, not the values at the point.

In the quotient rule, it's based on the limits, not the function values.

So it seems straightforward.

But the problem mentions that f is continuous, but for g, it doesn't say continuous, only that limit exists.

In the quotient rule, I don't think continuity of g is necessary, as long as the limit exists and is not zero.

For example, suppose g(x) has a removable discontinuity or something.

But still, the limit exists and is $2 \neq 0$.

So f/g should have limit 3.

But let me think of a counterexample.

Suppose f(x) = 6 for all x, which is continuous, limit at 2 is 6.

g(x) = 2 for $x \neq 2$, but g(2) = 0, for example.

Then lim g(x) as $x\to 2$ is 2, since for $x\neq 2$, g(x)=2.

Now, f(x)/g(x) = 6/g(x) for $x\neq 2$, which is 6/2 = 3 for $x\neq 2$, so limit is 3.

At x=2, g(2)=0, so f(2)/g(2) is undefined, but since we're taking limit as x approaches 2, we don't care about x=2.

So the limit exists and is 3.

Now, what if g(x) approaches 2 but oscillates or something.

But since limit is 2, it doesn't oscillate in a way that prevents the limit.

For example, g(x) = 2 + sin(1/(x-2)) * something, but if the limit is 2, it must be that the oscillation amplitude goes to zero.

Suppose g(x) = 2 + (x-2) * sin(1/(x-2)), then as x→2, (x-2)→0, and sin(whatever) is bounded, so g(x)→2.

Then f(x)/g(x), let f(x)=6.

Then f/g = 6 / (2 + (x-2)sin(1/(x-2)))

As x→2, the denominator →2, so f/g → 6/2=3.

And at points where denominator is large or small, but since (x-2) is small, the term (x-2)sin(...) is small, so denominator is close to 2.

For instance, | (x-2)sin(1/(x-2)) | ≤ |x-2|, which →0, so |g(x) - 2| ≤ |x-2|, so g(x) →2.

Then f/g → 6/2=3.

So seems fine.

What if g(x) is zero at some points near 2, but not all points.

But earlier I thought that since lim g(x)=2≠0, g(x) cannot be zero near 2 except possibly at x=2.

Yes, as I said, for |x-2| small, g(x) close to 2, so not zero.

Specifically, as above, with ε=1, |g(x)-2|<1 for |x-2|<δ, so g(x) between 1 and 3, not zero.

So no problem.

Perhaps if the limit of g is 0, but here it's 2, not zero.

In this case, it should be fine.

But the problem is to divide f/g, and if g approaches 0, it might blow up, but here g approaches 2≠0.

So why does the problem mention that f is continuous? It might not be necessary for the quotient rule.

But in this case, since f has a limit, and it's continuous, but limit exists anyway.

Perhaps to ensure that f(2) is defined or something, but for the ratio, we don't care about x=2.

Unless the question is about the value at x=2, but no, it says limit as x goes to 2.

"limit f(x)/g(x) as x goes to 2"

So it's the limit, not the value.

And f and g are both defined at 2, but f/g might not be defined at 2 if g(2)=0.

But for the limit, it's fine.

In my earlier example, if g(2)=0, then f(2)/g(2) is undefined, but the limit still exists.

So no issue.

Perhaps the problem is that if g(2)=0, and f(2)=6, but since we're taking limit, it doesn't matter.

I think I'm overcomplicating.

Maybe there's a case where g(x) approaches 2, but f(x) has some behavior.

But f is continuous, so it's well-behaved.

Suppose f is not continuous at 2, but in this case it is.

The problem says f is continuous, so its limit exists and equals f(2).

But for the ratio, I don't think it matters.

Perhaps if g(x) is very small at some points, but again, since lim g=2, it can't be too small near 2.

I think it's always true.

But let me read the problem again.

"There is a function f(x) which is continuous on all real numbers, and its limit is 6 as x goes to 2." This is a bit redundant; since it's continuous everywhere, the limit exists and equals the function value, so f(2)=6.

"another function g(x) which is defined on all real numbers, and its limit is 2 as x goes to 2."

So g(2) may not be 2.

Now, f(x)/g(x), as $x \to 2$.

Since $g(x) \to 2 \neq 0$, and $f(x) \to 6$, so $f(x)/g(x) \to 6/2 = 3$.

By the properties of limits.

I think it's always true.

But the question is "is it always true", and I think yes.

But perhaps there's a trick with g being zero at some points, but as discussed, not near 2 except possibly at 2.

Or if g has essential discontinuity, but limit still exists, so it must not oscillate too much.

Since limit exists, it means g approaches 2, so the values are controlled.

Perhaps if g is complex-valued or something, but no, real functions I assume.

The problem says "real numbers", so real functions.

I think I'm missing something.

Another thing: what if at x=2, g(2)=0, and f(2)=6, so f/g is undefined at 2, but the limit as x approaches 2 still exists and is 3, as in my example.

So for the limit, it's fine.

If the question were about the function f/g being defined or continuous at 2, that might be an issue, but it's not; it's about the limit.

So I think it's always true.

But let me see the way the question is phrased: "Is it always true that limit $f(x)/g(x)$ is 3 as x goes to 2?"

Based on standard limit theorems, yes.

Perhaps the problem is that g might be zero at points arbitrarily close to 2, but if $\lim g(x) = 2 \neq 0$, that can't happen.

Because if for every $\delta>0$, there is x with $|x-2|<\delta$ and $g(x)=0$, then $|g(x) - 2| = |0 - 2| = 2$, so if I take $\varepsilon=1$, $|g(x)-2| = 2 >1$, so it cannot be that $\lim g(x) =2$ if $g(x)$ is zero at points arbitrarily close to 2.

So indeed, since $\lim g(x)=2\neq0$, there exists some interval around 2 where $g(x) \neq 0$, and $|g(x)| >$ some positive number.

Thus f/g is defined near 2 except possibly at 2, and the limit exists.

So I think it's always true.

But why mention that f is continuous? Maybe to distract or something.

Perhaps if f wasn't continuous, but it is.

Another thought: what if $g(x)$ approaches 2 from below or something, but still, since it approaches 2, the ratio should approach 3.

I think I should conclude that it is always true.

But let me see if there is any counterexample.

Suppose $f(x) = 6$ for all x.

$g(x) = 2$ for x rational, $x\neq2$, and $g(x) = x$ for x irrational, or something.

But then what is $\lim g(x)$ as $x\to2$?

For $x\to2$, if x rational, $g(x)=2$, if x irrational, $g(x)=x\to2$, so in both cases, as $x\to2$, $g(x)\to2$

No problem.

If g has a different definition, but as long as it approaches 2.

So I think it's fine.

Perhaps the issue is with the domain; g is defined everywhere, so no holes.

I think I'm spending too much time; probably it is always true.

So the answer should be yes.

> But the problem is from a context where maybe they want to say no for some reason.
>
> Perhaps if the limit of g is zero, but here it's not.
>
> Or if f and g both approach zero, but here both limits are non-zero.
>
> 6 and 2.
>
> So I think it's safe.
>
> I'll go with yes, it is always true.

The limit of $f(x)/g(x)$ as x approaches 2 is always 3 under the given conditions. This follows from the properties of limits and the given information.

Since $f(x)$ is continuous on all real numbers and its limit as x approaches 2 is 6, $f(2) = 6$.

The limit of $g(x)$ as x approaches 2 is 2, and $g(x)$ is defined on all real numbers, but g may not be continuous at 2, so $g(2)$ may not equal 2.

To find the limit of $f(x)/g(x)$ as x approaches 2:

The limit of $f(x)$ as $x \to 2$ is 6.

The limit of $g(x)$ as $x \to 2$ is 2, and since $2 \neq 0$, the quotient rule for limits applies.

The quotient rule states that if $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = M$ with $M \neq 0$, then $\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{L}{M}$.

Here, $L = 6$ and $M = 2$, so $\lim_{x\to 2} \frac{f(x)}{g(x)} = \frac{6}{2} = 3$.

Additionally, since $\lim_{x\to 2} g(x) = 2 \neq 0$, there exists a neighborhood around $x = 2$ (excluding possibly $x = 2$ itself) where $g(x) \neq 0$. This ensures that $f(x)/g(x)$ is defined near $x = 2$, and the limit exists.

The continuity of $f(x)$ ensures that its limit exists and equals $f(2)$, but it is not necessary for the application of the quotient rule; only the existence of the limits and $M \neq 0$ are required.

Thus, it is always true that $\lim_{x\to 2} \frac{f(x)}{g(x)} = 3$.

\boxed{\text{yes}}

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