k is a positive real number and the equation of a hyperbola is x^2 / (k + 1)^2 - y^2 / (k - 1)^2 = 1. If the coordinates of focii are F(2*sqrt(13), 0) and F'(-2*sqrt(13), 0), what is the equation of this hyperbola?

Answer from Mathful

Thought for 100 seconds

The problem involves a hyperbola given by $\frac{x^2}{(k+1)^2} - \frac{y^2}{(k-1)^2} = 1$, with foci at $F(2\sqrt{13}, 0)$ and $F'(-2\sqrt{13}, 0)$. I need to find the equation of the hyperbola, which means I need to find the value of k.

First, recall that for a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the foci are at $(\pm c, 0)$, where $c = \sqrt{a^2 + b^2}$.

In this equation, $a^2 = (k+1)^2$ and $b^2 = (k-1)^2$. Since k is a positive real number, and assuming $k > 1$ because otherwise b^2 might be negative or zero, which doesn't make sense for a hyperbola. Let me check that.

If $k < 1$, then $k - 1 < 0$, so $(k-1)^2$ is still positive, so it might be okay, but let's see the foci. The foci are given on the x-axis, so it's a standard hyperbola opening left and right, so a and b should be real and positive, which they are since squared. But for the hyperbola to be defined, a and b need to be positive, which they are as long as $k \neq -1$ or 1, but since k is positive real, it should be fine. However, if $k < 1$, $b^2 = (k-1)^2$ is still positive, but let me see the distance to foci.

The distance from center to each focus is c, and here $c = 2\sqrt{13}$, since foci are at $(\pm 2\sqrt{13}, 0)$.

So, $c = 2\sqrt{13}$.

For a hyperbola, $c^2 = a^2 + b^2$.

So, in this case, $a^2 = (k+1)^2$, $b^2 = (k-1)^2$.

So, $c^2 = a^2 + b^2 = (k+1)^2 + (k-1)^2$.

And $c^2 = (2\sqrt{13})^2 = 4 \times 13 = 52$.

So, I can write:

$(k+1)^2 + (k-1)^2 = 52.$

Let me expand this.

$(k+1)^2 = k^2 + 2k + 1$

$(k-1)^2 = k^2 - 2k + 1$

Add them: $k^2 + 2k + 1 + k^2 - 2k + 1 = 2k^2 + 2$

Set equal to 52:

$2k^2 + 2 = 52$

Subtract 2: $2k^2 = 50$

Divide by 2: $k^2 = 25$

So, $k = 5$ or $k = -5$

But k is a positive real number, so $k = 5$.

Now, I should verify if this makes sense.

If $k = 5$, then $a^2 = (5+1)^2 = 6^2 = 36$

$b^2 = (5-1)^2 = 4^2 = 16$

Then $c^2 = a^2 + b^2 = 36 + 16 = 52$, and $c = \sqrt{52} = 2\sqrt{13}$, which matches the foci.

Now, is there any issue with $k < 1$? Earlier I got only $k = 5$ as positive, since -5 is negative.

But what if $k = 1$? If $k = 1$, then $b^2 = (1-1)^2 = 0$, and the equation becomes $\frac{x^2}{(1+1)^2} - \frac{y^2}{0} = 1$, which is undefined, and not a hyperbola. Similarly, if $k = -1$, same issue. But since k is positive, we're good.

The problem says "k is a positive real number", and I got k=5, which is fine.

But let me see the equation. With k=5, a^2=36, b^2=16, so equation is $\frac{x^2}{36} - \frac{y^2}{16} = 1$.

The foci are at $(\pm c, 0) = (\pm\sqrt{36+16}, 0) = (\pm\sqrt{52}, 0) = (\pm 2\sqrt{13}, 0)$, which matches.

The problem asks for the equation, not k. It says "what is the equation of this hyperbola?"

So, with k=5, the equation is $\frac{x^2}{(5+1)^2} - \frac{y^2}{(5-1)^2} = 1$, which is $\frac{x^2}{36} - \frac{y^2}{16} = 1$.

I should write it in standard form.

Is there any other consideration? Earlier I assumed that a^2 = (k+1)^2 and b^2 = (k-1)^2, but is it possible that it's the other way around? For example, if k<1, but in this case, if k<1, say k=0.5, but we got k=5 as solution, and k=-5 not positive.

In the equation, a and b are both squared, so it might be that (k+1)^2 is not necessarily larger than (k-1)^2. In this hyperbola, since it's opening left-right, a should be the distance to vertex, so typically a > b, but actually, for hyperbola, a and b are just parameters, and c^2 = a^2 + b^2 always, but a is associated with the transverse axis.

In this case, when k=5, a^2=36, b^2=16, so a=6, b=4, fine.

But suppose k<1, say if k were less than 1, but in our equation, we have (k+1)^2 + (k-1)^2 =52, which gave k=±5, so only solutions, no other.

But just to be thorough, suppose the hyperbola was oriented differently, but the foci are on x-axis, so it should be fine.

Another thing: in the equation, it's written as $\frac{x^2}{(k+1)^2} - \frac{y^2}{(k-1)^2} = 1$, which suggests that (k+1)^2 is a^2, but if k<1, k+1 could be less than |k-1|, but since it's squared, it's always positive.

For example, if k=0, then a^2=(1)^2=1, b^2=(-1)^2=1, so c^2=1+1=2, but c=√2, not 2√13.

With k=5, it works.

But in our solution, k=5 is greater than 1, so no issue.

The problem is to find the equation, so with k=5, it's $\frac{x^2}{36} - \frac{y^2}{16} = 1$.

I can write it as $\frac{x^2}{6^2} - \frac{y^2}{4^2} = 1$, but usually we write with denominators.

Now, is this simplified? I think so.

But let me check if the foci are correctly given. F(2√13, 0) and F'(-2√13, 0), and distance between foci is 4√13, and for hyperbola, distance between foci is 2c, so 2c=4√13? No.

c is distance from center to focus, so distance between foci is 2c.

Given foci at ±2√13, so distance between them is 4√13, so 2c = 4√13, thus c=2√13, as we had.

c^2=52, a^2 + b^2=36+16=52, good.

Now, just to be sure, is there any other hyperbola that could satisfy this? For example, if it were vertical, but the foci are on x-axis, and the equation has x^2 positive, so it must be horizontal transverse axis.

The general equation could have a and b switched, but in this case, since it's written with x^2 over (k+1)^2 and y^2 over (k-1)^2, and for the hyperbola to open left-right, the coefficient of x^2 should be positive, and denominator larger, but in this case, with k=5, (k+1)^2=36, (k-1)^2=16, so 36>16, so it's fine.

But suppose k<1, say if k were such that (k-1)^2 > (k+1)^2, but in that case, if I plug into the equation, but in our solution, no such k satisfies the foci condition.

For example, if k=0, a^2=1, b^2=1, c^2=2, not 52.

Or k=2, a^2=9, b^2=1, c^2=10, not 52.

k=5 is the only positive solution.

But earlier I thought about k<1, but no solution there.

Another way to think: in the equation (k+1)^2 + (k-1)^2 =52, which simplifies to 2k^2 + 2 =52, so k^2=25, k=5 or -5, only positive is 5.

So, equation is $\frac{x^2}{36} - \frac{y^2}{16} = 1.$

I can write it as $\frac{x^2}{6^2} - \frac{y^2}{4^2} = 1$, but usually we leave it as fractions.

Sometimes they simplify, but 36 and 16 are both squares, but I think it's fine.

I can check with a point, but probably unnecessary.

The problem says "the coordinates of focii are", I think it's a typo, should be "foci", but anyway.

So, I think that's the answer.

But let me see if the equation is correct.

When x=6, $\frac{36}{36} - \frac{y^2}{16} = 1$, so 1 - y^2/16 =1, thus y=0, so vertices at (±6,0).

Foci at (±2√13, 0) ≈ (±7.211, 0), and since c>a, 2√13 ≈ 2*3.606=7.212, a=6, so distance from center to focus is about 7.212, which is greater than a=6, good for hyperbola.

c^2 = a^2 + b^2 = 36 + 16 =52, correct.

So, the equation is $\frac{x^2}{36} - \frac{y^2}{16} = 1.$

Now, to box the answer.

But the problem says "what is the equation", so I should write it clearly.

I could write it without fractions, but I think this is standard.

Since it's given in terms of k, but I think they want the numerical equation.

So, I'll box that.

But let me confirm the calculation again.

c = 2√13, so c^2 = 4 * 13 = 52.

a^2 = (k+1)^2, b^2 = (k-1)^2.

a^2 + b^2 = (k+1)^2 + (k-1)^2 = k^2 + 2k + 1 + k^2 - 2k + 1 = 2k^2 + 2.

Set to 52: 2k^2 + 2 = 52 → 2k^2 = 50 → k^2 = 25 → k=5 (since positive).

Then a^2 = (5+1)^2 = 6^2 = 36.

b^2 = (5-1)^2 = 4^2 = 16.

Equation: $\frac{x^2}{36} - \frac{y^2}{16} = 1.$

I could write it as $\frac{x^2}{6^2} - \frac{y^2}{4^2} = 1$, but same thing.

I think it's fine.

So, the equation is $\frac{x^2}{36} - \frac{y^2}{16} = 1.$

Now, for the box.

> But in the problem, the original equation has denominators as squares, so probably they expect the same form.
>
> I could simplify by dividing numerator and denominator, but no, it's already simplified.
>
> Note that 36 and 16 have a common factor of 4, but in hyperbola equations, we usually leave it as is, since it's standard form.
>
> So, I'll go with that.

The hyperbola is given by $\frac{x^2}{(k+1)^2} - \frac{y^2}{(k-1)^2} = 1$, with foci at $F(2\sqrt{13}, 0)$ and $F'(-2\sqrt{13}, 0)$.

For a hyperbola of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the foci are at $(\pm c, 0)$, where $c^2 = a^2 + b^2$. Here, $a^2 = (k+1)^2$ and $b^2 = (k-1)^2$.

The distance from the center to each focus is $c = 2\sqrt{13}$, so:

$$c^2 = (2\sqrt{13})^2 = 4 \cdot 13 = 52.$$

Thus:

$$a^2 + b^2 = (k+1)^2 + (k-1)^2 = 52.$$

Expanding the left side:

$$(k+1)^2 + (k-1)^2 = (k^2+2k+1) + (k^2-2k+1) = 2k^2+2$$

$$2k^2 + 2 = 52.$$

Solving for k:

$$2k^2 = 50, \quad k^2 = 25, \quad k = \pm 5.$$

Since k is a positive real number, $k = 5$.

Now, substituting $k = 5$:

$$a^2 = (5+1)^2 = 6^2 = 36, \quad b^2 = (5-1)^2 = 4^2 = 16.$$

Thus, the equation of the hyperbola is:

$$\frac{x^2}{36} - \frac{y^2}{16} = 1.$$

Verification:

The foci are at $(\pm\sqrt{36+16}, 0) = (\pm\sqrt{52}, 0) = (\pm 2\sqrt{13}, 0)$, which matches the given foci.

For $k = 5$, $k > 1$, so the denominators are positive and the hyperbola is well-defined.

\boxed{\dfrac{x^{2}}{36} - \dfrac{y^{2}}{16} = 1}

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